A case study of the application of spectral analysis of very low frequency (VLF) dip angle measurements in Arigidi, Akoko area, southwestern Nigeria

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Abstract

Very low frequency electromagnetic (VLF-EM) fields caused by geologic bodies such as faults, fissures, and fractures have been modeled as resulting from simple subsurface geometric, electric conductors namely, lines, sheets, and tabular plates and their VLF-EM response computed using the Biot-Savart law of electromagnetism. The Fourier spectra of the VLF-EM fields of these geologic bodies were obtained as a sum of exponential functions which depend on the wave number (spatial frequency) and depth of burial of the bodies.

The results of the spectral analysis of the VLF-EM fields have been applied to VLF measurements in a hydro-geologic context of fracture location and depth computation in a basement complex terrain of Arigidi in Akoko area of southwestern Nigeria. The profile EM data were acquired using the GEONICS EM-16 VLF receiver.

The VLF measurements successfully located subsurface fractures in the direction of N 60° W. Spectral depth inversion to basement on the profiles yielded results that vary between 19.6m at the shallower end and 35.7m at the deeper end in the area of study. The results confirm that the VLF-EM methods are a fast and relatively low-cost method of weathered layer evaluation in the basement terrain.

Introduction

A time-varying electromagnetic field introduced on the surface of the ground, will produce secondary currents in subsurface conductors in accordance with the laws of electromagnetic induction. These currents give rise to secondary electromagnetic fields, which modify the total field observed at any point on the surface. Generally, the measured resultant field will differ from the primary field in intensity, phase and direction and reveal the presence of the subsurface conductors (Parasnis, 1994). The limitation of this approach is that the interpretation is restricted to single specific models.

In this paper, a novel approach to the interpretation of VLF data is presented from an investigation on a typical basement terrain, specifically around Arigidi in southwestern Nigeria (Fig. 1). The edges of subsurface geological bodies are modeled as simple geometric conductors as lines, planar sheets and tabular plates. The transmitter geologic target configuration are governed by the following assumptions:

(a) the field of the electric dipole of the VLF transmitter is uniform within a small area located at large distance from the transmitter.
(b) The VLF ground wave arrives in approximately the geologic strike direction in the area of measurement; (Telford et al., 1990)
(c) The models are horizontal thin sheet/tabular plates and linear conductors which approximate to geologic sand/weathered basement and fracture zones. (Telford et al., 1990).

Spectral response of VLF - EM signals due to electric lines of current

Theory

It is assumed that VLF measurements are made along a profile OX. A nomal field observed in this direction arises from concentrated (or gathered) and induced line currents flowing along perpendicular direction (OY) to the plane of measurement (Fig. 2a)

In the two - dimensional case (Fig. 2b), let I(U,V) be the current density at location (U,V) of element, I dudv. Then the vertical component H (x), of the magnetic field caused by the current density, I (U,V) is given by the application of
Fig. 1. Geological Map of Arigidi, Southwestern Nigeria showing area of study

Fig. 2. Schematic diagram of VLF measurement configuration (b) Observed anomalous electromagnetic field $H_z$ due to current density $I$ at depth $Z = V$. 
Biot-Savart law (Reitz and Milford, 1996 pp., 154) as follows:

\[ H_2(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} l(u,v) \frac{(x-u)du}{[(x-u)^2 + v^2]} \]  

(1a)

Now for currents concentrated in a ribbon at some depth \( Z = V \), as shown in Fig. 2, equation (1a) may be written as

\[ H_2(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} l(u,v) \frac{(x-u)du}{[(x-u)^2 + v^2]} \]  

(1b)

At this depth level, the current density function \( l(U,v) \) is only a function of \( U \). Denote this function by \( l_v(u) \).

Therefore, equation (1b) can be written as

\[ H_2(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} l_v(u) \frac{(x-u)du}{[(x-u)^2 + v^2]} \]  

(2)

The second integral on the right hand side of equation (2) is convolution integral (Bracewell, 1965, pp. 25). Therefore equation (2) can be written as:

\[ H_2(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} l_v(u) \frac{(x-u)du}{[(x-u)^2 + v^2]} \]  

(3)

where

\[ l_v(x) = l_v(u) \]  

\[ G_v(x) = \frac{x}{[x^2 + V^2]} \]  

(4)

and \( * \) denotes convolution.

Since the current density is restricted to infinitesimal ribbon at depth \( Z=V \), equation (3) can be evaluated as

\[ H_2(x) \bigg|_{z=\varepsilon} = \frac{\Delta V}{2\pi} \left[ l_v(x) G_v(x) \right] \]  

(5)

Fourier transforming equation (5) (Bracewell, 1965, pp. 110) leads to:

\[ \hat{H}_2(k) \bigg|_{z=\varepsilon} = \frac{\Delta V}{2\pi} \left[ \hat{l}_v(K) \hat{G}_v(K) \right] \]  

(6)

Where \( \hat{H}_2(K), \hat{l}_v(K), \hat{G}_v(K) \) are respectively the Fourier transforms of \( H_2(x) \) \( l_v(x) \) and \( G_v(x) \); \( K \) is the wave number in the x-direction. It has also been assumed that the Fourier transform of the current density function \( l_v(x) \) exists. That is,

\[ \hat{l}_v(K) = \int_{-\infty}^{\infty} l_v(x)e^{ix} dx < \infty \]  

(7)

From equation (4)

\[ G_v(x) = \frac{x}{(x^2 + V^2)} \]  

(7)

The Fourier transform of \( G_v(x) \) can be obtained by using Erdelyi et al (1954 no. 3.2.5) to give:

\[ \hat{G}_v(K) = -i e^{\varepsilon |K|} \]  

(8)

The current density function \( l_v(x) \) is not known. However, some reasonable statistical assumption can be made about this function. If the current density values vary randomly and are uncorrelated, then we have

\[ E[l_v(x)l_v(x+\tau)] = \sigma^2 \delta(r) \]  

(9)

Where \( E \) denotes the expected value and \( \sigma^2 \) is the kronecker delta (Bracewell, 1965), \( \sigma^2 \) is the power of the current density function. The power spectrum, \( S_{1h} \) of \( l_v(x) \) is now given by

\[ S_{1h} = \left( \frac{\Delta V}{2\pi} \right)^2 \left[ l_v(x) \right] \hat{l}_v(K) \hat{l}_v(K) = \sigma^2 \]  

(10)

Where * denotes complex conjugation. Using equation (6) above, the power spectrum, \( S_{1h} \), of the observed measurements is given by

\[ S_{1h}(K,V) = \left( \frac{\Delta V}{2\pi} \right)^2 \left[ l_v(x) \hat{l}_v(K) \hat{l}_v(K) \hat{G}_v(K) \hat{G}_v(K) \right] \]  

(11a)

Where * denotes complex conjugation. That is,

\[ S_{1h}(K,V) = \left( \frac{\Delta V}{2\pi} \right)^2 \left[ l_v(x) \hat{G}_v(K) \hat{G}_v(K) \right] \]  

(11b)

Where \( S_{1h}(k) \) is the spectrum of \( G(x) \). Substitution of equations (8) and (10) into equation (11b) leads to

\[ S_{1h}(K,V) = \left( \frac{\Delta V}{2\pi} \right)^2 \left[ l_v(x) \hat{G}_v(K) \hat{G}_v(K) \right] \]  

(12a)

\[ S_{1h}(K,V) = A_1 e^{-2V_1 |K|} + A_2 e^{-2V_2 |K|} \]  

(12b)

\[ A_1 = \left( \frac{\Delta V}{2\pi} \right)^2 \sigma^2 \]  

\[ A_2 = \left( \frac{\Delta V}{2\pi} \right)^2 \sigma^2 \]  

\[ S_{1h}(K,V) = \text{normalized } S_{1h}(K,V) \]  

(13)

For \( \sigma_1 = \sigma_2 = A_1 = A_2 = A \)

Therefore

\[ S_{1h}(K,V) = \frac{1}{2} e^{-2V_1 |K|} + \frac{1}{2} e^{-2V_2 |K|} \]  

(13)

In general

\[ S_{1h}(K,V) = \left( \frac{A_1 + A_2}{2} \right) e^{-2V_1 |K|} + \left( \frac{A_1 + A_2}{2} \right) e^{-2V_2 |K|} \]  

(14)

\[ S_{1h}(K,V) = \left( \frac{1}{1+c^2} \right) e^{-2V_1 |K|} + \left[ \frac{1}{1+(c^2)^2} \right] e^{-2V_2 |K|} \]  

(15)
where \( C = \frac{\sigma_0}{\sigma_1} \)

**Computation of the power spectrum of the field data**

Let the observed field data be discrete space series.

\[
H(x) = [h_0, h_1, h_2, h_3, \ldots, h_j]
\] (16)

Observed at equispaced distances:

\[
x = i \Delta x, \ i = 0,1,2, \ldots, J,
\] (17)

A long a profile, the power spectrum of \( H(x) \) can be estimated from either (i) the autocorrelation of \( H(x) \) or (ii) from the wave number frequency spectrum (periodogram) of \( H(x) \) [Davies et al., 1973]. We shall employ the second method here. To compute the frequency spectrum, it is necessary first to eliminate any large discontinuity that may occur at the beginning and end of data vector \( H(x) \). This is achieved by applying a taper to the recorded information so that the first and last data points approach the mean value, which is usually zero. Kanasewich (1975) has suggested a taper function, \( W \), consisting of a pair of cosine bells such that for \( J \) data points, each \( n \)-th component is multiplied by:

\[
W_n = \begin{cases} 
\frac{1}{2} (1 + \cos \frac{\pi n \Delta x}{M}) & \text{ if } L < n < L + M \\
\frac{1}{2} (1 + \cos \frac{\pi n \Delta x}{M}) & \text{ if } L < n < L + M
\end{cases}
\] (18)

Where \( M \) determines the period of the cosine bell and \( 2L \) is the number of unweighted data points.

The next step is to compute the wave number spectrum from weighted \( H(x) \). To do this, a number of trailing zeroes are added to weighted \( H(x) \) until the total number of points equals \( N \) such that \( N = 2^m \) where \( m \) is any integer, and \( N >>m \). A fast Fourier transform algorithm (Oppenheim and Schafer, 1975) can now be applied to modified \( H(x) \) to obtain the wave number spectrum \( \hat{H}(l) \) such that

\[
\hat{H}(l) = \sum_{n=0}^{N-1} h(n)e^{-j \frac{2\pi n l}{N}} \quad L = 0,1,2, \ldots, N - 1
\] (19)

Now the power spectrum \( S_m(l,Z) \) of \( H(x) \) is given by

\[
S_m(l,Z) = \hat{H}_m(l) \hat{H}^*_m(l)
\] (20)

Where \( m \) is the order of the harmonic, \((*)\) denotes complex conjugation and the subscript, \( z \), has been employed to emphasize the depth dependence of these functions. The wave number \( k \) is now discrete and is given by

\[
k_l = \frac{2\pi l}{(N - 1) \Delta x} \quad l = 0,1,2, \ldots, N - 1
\] (21)

**Depth computation**

The computation of depth to the horizontal conductors underlying the VLF profile is carried out by using equations (13) or (A 4) in conjunction with equations (14) and (21) respectively, (Table 1). We have established the gradient, \( m \), of the log-linear graph as [see equation (14)]

\[
m = 2v = \frac{\Delta \ln [S_m(K,Z)]}{\Delta K}
\]

Where \( S_m(k,z) \) is the power spectrum, \( k \) the wave number, and \( v \), the depth to the conductor. Now from equation (21), \( \Delta K \) is given as

\[
\Delta K = \frac{2\pi \Delta l}{(N - 1) \Delta x}
\]

Where

\( N \) = number of extended data points used in computing the spectrum

\( \Delta X \) = Incremental spatial distance of the observation points (spatial sampling in sampling distance of the observation points)

\( l \) = Harmonic numbers

Therefore gradient \( m \) is given as

\[
m = 2v = \frac{(N - 1) \Delta X \Delta \ln [S_m(K,Z)]}{2\pi \Delta l}
\] (22)

and depth, \( V \), is given by

\[
V = \frac{-(N - 1) \Delta X \Delta \ln [S_m(K,Z)]}{4\pi \Delta l}
\] (23)

When the spectrum is plotted on a log_{10} - linear graph, the equation for depth, \( V \), becomes

\[
V = \frac{-(N - 1) \Delta X \Delta \log_{10} [S_m(K,Z)]}{4\pi \log_{10}(e) \Delta l}
\] (24)

Where

\( e \) = root of natural logarithm (and equals 2.71828)

W hen the values of the spectrum \( S_m(l, i = 1, 2 \ldots) \) and the corresponding harmonics \( l, i = 1, 2 \) are read from the straight line graph, the value of depth to the conductor is obtained as

\[
V = \frac{-(N - 1) \Delta X \Delta \log_{10} [S_m(K,Z)]}{4\pi \log_{10}(e) \Delta l}
\] (25)

**Case history from Arigidi, southwestern Nigeria**

The geology of Arigidi, Southwestern Nigeria has been reviewed by Rahman (1975). A rigid area is underlain by the basement complex rocks. The main geological unit in this area being the M ignate-gneiss complex illustrated in Fig. 1, which was adapted from the geological survey of Nigeria (GSN). In this figure, there exist two main structural
Trends in the Arigidi area. An east-west linear trend occurs in the south while in the north, the trend is in the northwest direction.

Rainfall in this area is very scanty and seasonal, namely, wet and dry. In the dry season, rainfall is very sparse about 250-500 mm. Thus, the main water supply is derived from low yield hand-dug wells. Geophysical techniques targeting areas of intense bedrock weathering and or fractures become applicable for the location of productive aquifers in both domestic and industrial use.

VLF-EM Measurements

The VLF-EM method is based on the measurement of the secondary magnetic field induced in local conductors by the primary EM fields generated by powerful naval radio transmitters in the very low frequency range (15 – 30 kHz). The instrument employed for the survey was the GEONICS EM-16 Receiver. This instrument measures the induced vertical magnetic field as a percentage of the horizontal primary field (GEONICS, 1979). At Arigidi, the VLF-EM measurements were observed at station intervals of 10m along each of two profiles P1, P2 respectively. These profiles lie west-east along traverse TR-9 and southwest-northeast along TR-11 respectively. The operational VLF transmitting stations at the time of the survey were GBR located in Rugby, England, and FUO, located in Bordeaux, France.

Data Interpretation

Fig. 3 shows profiles P1 and P2 observed in VLF survey at Arigidi. In this case, the objective is the location of sand/weathered basement and fracture zones, which have been respectively modeled as horizontal thin sheet or tabular plate and linear conductors.

The VLF-EM measurements on profiles P1 and P2 obtained at Arigidi are shown in Fig 3a and 3b. Examination of the signatures show that they consist of peaks and troughs. A peak occurs at station number 8 on P1, traverse TR-9. Three peaks occur at station numbers 7, 14 and 20 respectively on profile P2 on traverse TR-11. These peaks are qualitatively indicative of fracture occurrence. Spatial correlation of these peaks on the two profiles P1, P2 respectively on traverses TR-9 and TR-11 yielded a direction of N60 W confirming the geologic linear in the northern part of Arigidi. The depth to these fractures can now be determined by application of the spectral technique developed in the foregoing.

Profiles P1 and P2 (Fig. 3a and b) were observed at regular spatial sampling increment X = 10 metres and each having data vector lengths of 13 and 21 points respectively. The recorded information was weighted using equation (18) and the length of the data record extended by trailing zeros to N=128 points, that is (N=22), in each case. The power spectrum for each profile was then computed using the Fast Fourier Transform (FFT) as shown under the foregoing section on computation of power spectrum.

Figs. 4 and 5 show respectively the power spectra on profiles P1 and P2 plotted on semilogarithmic (log10 – linear) graph. A close examination of the figures reveals that the power spectrum consists of an exponential function of the form of equation (12) modulated by a sine function whose argument is related to the extent of the target width. Thus the power spectrum appears a series of peaks and troughs. The straight line of equation (12) or segmented straight lines can be fitted to power spectrum by aligning the peaks of the power spectrum (Figs. 4 and 5).

Fig. 4 shows the power spectral plot of profile P1. It consists of a noise level and two segments I and II representing two horizontal conductors respectively. Depth to a conductor is computed from the corresponding plot segment by reading off the values of the spectrum (S1, S2) at two convenient harmonic values (L1, L2) respectively and using equation (25) to calculate the depth. Table 2A shows the spectral values and harmonics obtained for the two conductors of Fig. 4. Depths computed for the two sources are 18.4 and 19.6 metres respectively.

Table 1. Table of equation (13) for:

(a) One current sheet at depth 10m

<table>
<thead>
<tr>
<th>K</th>
<th>-20k</th>
<th>S_1/Z_1 = 10m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.05</td>
<td>-1.0</td>
<td>0.3679</td>
</tr>
<tr>
<td>0.10</td>
<td>-2.0</td>
<td>0.1353</td>
</tr>
<tr>
<td>0.15</td>
<td>-3.0</td>
<td>0.0498</td>
</tr>
<tr>
<td>0.20</td>
<td>-4.0</td>
<td>0.0183</td>
</tr>
<tr>
<td>0.25</td>
<td>-5.0</td>
<td>0.00674</td>
</tr>
<tr>
<td>0.30</td>
<td>-6.0</td>
<td>0.00248</td>
</tr>
<tr>
<td>0.35</td>
<td>-7.0</td>
<td>0.00091</td>
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<tr>
<td>0.40</td>
<td>-8.0</td>
<td>0.00034</td>
</tr>
<tr>
<td>0.45</td>
<td>-9.0</td>
<td>0.00012</td>
</tr>
<tr>
<td>0.50</td>
<td>-10.0</td>
<td>0.000045</td>
</tr>
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</table>

(b) Two current sheets at depths 5m and 20m

<table>
<thead>
<tr>
<th>-10k</th>
<th>S_2/Z_2 = 5m</th>
<th>-40k</th>
<th>S_1/Z_1 = 20m</th>
<th>S_{mean} = \frac{S_1+S_2}{2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>-0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
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<td>-1.0</td>
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<td>-4.0</td>
<td>0.0183</td>
<td>0.1931</td>
</tr>
<tr>
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<tr>
<td>-2.5</td>
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</tr>
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<td>0.00674</td>
<td>-20.0</td>
<td>0.000000002</td>
<td>0.00337</td>
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</table>
Fig. 3. Plot of normalized power spectrum as function of wave number (a) current sheet buried at 10m (b) currents sheets buried at 5m and 20m respectively.

<table>
<thead>
<tr>
<th>Model</th>
<th>Current Depth</th>
<th>Power Spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10m</td>
<td>$S(K_n) = e^{-20K}$</td>
</tr>
<tr>
<td>1</td>
<td>5m, 20m</td>
<td>$S(K_n) = 0.5e^{-10K}$</td>
</tr>
</tbody>
</table>

Fig. 4. VLF vertical magnetic field measurements (Real Component) on Profile (a) TR-9 (b) TR-11.
Table 2. Depth interpretation of power spectral plots

(A) Depth interpretation of power spectral plot in profile TR – 9

<table>
<thead>
<tr>
<th>Profile</th>
<th>P1: N = 128, AX = 10m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment</td>
<td>$I_1$, $I_2$, $S_1$, $Y_1=\log_{10}(S_1)$, $S_2$, $Y_2=\log_{10}(S_2)$, Depth(m)</td>
</tr>
<tr>
<td>I</td>
<td>14, 36, 0.425, 0.0425, 18.4</td>
</tr>
<tr>
<td>II</td>
<td>0, 16, 6.5, 0.29, 19.6</td>
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</tbody>
</table>

(B) Depth interpretation of power spectral plot in profile TR – 11

<table>
<thead>
<tr>
<th>Profile</th>
<th>P2: N = 128, X = 10m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment</td>
<td>$I_1$, $I_2$, $S_1$, $Y_1=\log_{10}(S_1)$, $S_2$, $Y_2=\log_{10}(S_2)$, Depth(m)</td>
</tr>
<tr>
<td>I</td>
<td>9.2, 33, 1.0, 0.0000, 0.07, 11.29</td>
</tr>
<tr>
<td>II</td>
<td>0, 10, 24, 0.7, 35.72</td>
</tr>
</tbody>
</table>

Fig. 5. Plot of normalized power spectrum as function of Harmonic numbers on profile TR-9
The power spectral plot of profile P2 (Fig. 6) also consists of a noise level and two segments labeled I and II representing two conductors respectively. Spectral values (S1, S2) corresponding to harmonic values (L1, L2) were read from segments I and II as shown in Table 2b. The various depths were calculated using equation (25). Obtained depths were 11.3 and 35.7 metres respectively for segments I and II. Figure 7 shows interpreted geologic litho-section, P1, P2, across traverse TR-9 and TR-11.

Conclusions
The power spectrum of VLF measurements arising from line currents and horizontal sheets has been obtained using the Biot-Savart law and simple statistical assumptions. The assumed models represent realistic geologic features associated with rock contacts and faults encountered in ground water prospecting and mineral exploration, which are areas of wide economic interest. Depth to target subsurface conductor was calculated from the slope of the power spectrum – wave number plot. The developed technique of power spectral analysis of ground VLF-EM measurements is a useful and fast interpretation tool applicable in engineering geophysics and mineral exploration.

References


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